Standard Form

We use standard form whenever a number is too small or too large to write easily in the usual way - this is used all the time in economics, science & business.

1. Write 2,900,000,000,000$ in standard form. (This is the GDP of the UK)
2. Express 6.7 x 10⁷ as a normal number. (This is the UK’s population)
3. Calculate the GDP per capita (=’per person’) in the UK, giving your answer in standard form. If 1$ = £0.79, how much is this in £?
4. Express 6,000,000,000,000,000,000,000,000kg in standard form. (This is the mass of the Earth, M)
5. If the radius of the Earth is r = 6.4 x 10⁶m, calculate r².
6. If a falling object accelerates at

calculate the acceleration of objects falling at the Earth’s surface.

(This is the acceleration due to gravity - if you drop anything on the surface of the Earth, it always increases its velocity at a rate of 9.8 meters per second per second!)

1. If a star is 500 Light-Years away (1 Light-Year = m), calculate: a) the distance in meters, b) the distance in km, and c) the time taken for light from the star (speed = meters per second) to reach the Earth. If 1 year = , how many years does it take for the light to reach us?

In years:

(This is what a Light-Year is: 1Ly = the distance that light travels in 1 year. Thus, a star 500Light-Years away will emit light that takes exactly 500 years to reach us!)

1. The table below shows some data:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Planet** | **Radius** | **Mass** | **Volume** | **Time taken to orbit** | **Mean orbital distance** |
| **Earth** | 6370km | 6.0⨉10²⁴kg |  | 3.2⨉10⁷s | 1.0AU |
| **Jupiter** | 69,900km | 1.9⨉10²⁷kg |  | 3.7⨉10⁸s | 5.2AU |
| **Uranus** | 25,400km | 8.7⨉10²⁵kg |  | 2.6⨉10⁹s | 20AU |
| **Pluto** | 1190km | 1.3⨉10²²kg |  | 7.8⨉10⁹s | 49AU |

1. Convert the radius of each planet to metres, and give the answers in standard form.

Earth:

Jupiter:

Uranus:

Pluto:

1. Use the formula for the volume of a sphere, (r = radius) to calculate the volumes of each planet (in m³). Fill in the **Volumes** column with volumes in standard form.

Earth:

Jupiter:

Uranus:

Pluto:

1. Calculate the number of seconds in 1 year, and fill in the **Time taken to orbit** column.
2. How many Earth years does it take Jupiter, Uranus and Pluto to orbit the Sun?

(Time taken to orbit in seconds) ÷ (3.2⨉10⁷ seconds in 1 year)

Jupiter: 11.6 years

Uranus: 81.2 years

Pluto: 244 years

1. If 1AU = 1.5⨉10⁸km, calculate the mean orbital distance from the Sun for each planet in standard form

Earth:

Jupiter:

Uranus:

Pluto:

1. Assuming that the planets orbit in circles around the Sun, find the circumference () of the orbit of each planet. Give your answer in standard form.

Earth:

Jupiter:

Uranus:

Pluto:

1. Why is the value given the ‘mean’ orbital distance? What is wrong with our assumption in part (f)?

Planets actually move in ellipses, not in circles: an ellipse is like a circle that is longer in one direction than another. This means the orbital distance actually changes at different times in the orbit.

Geometry of Circles & Spheres

Circles and spheres are the simplest 2D and 3D shapes (since they are the same in all directions), and are therefore some of the most mathematically important.

Circle: area = , perimeter =

Sphere: volume = , surface area =

1. If a circle has a radius of 10cm, what is a) its perimeter, and b) its area?
2. If a sphere has a diameter of 18cm, what is a) its volume, and b) its surface area? What is the ratio between volume and surface area?

Ratio of V to A =

(This makes sense because . The ratio of the volume of a sphere to the surface area of the sphere is always )

1. What is an equation for the total surface area of a half-sphere?



Total surface area =

Surface area of curved face + Surface area of flat circle

1. If Pluto has a radius of m and Venus has a radius of m, calculate the volume & surface area of each planet. What is the ratio of volumes ()? What is the ratio of surface areas? What is the ratio of radii?

Pluto:

Venus:

Ratio of volumes (or 214 if rounded answers used)

Ratio of areas

Ratio of radii

(These are scale factors: if the radius increases by a factor of 6, then the area increases by a factor of 6x6=36, and the volume increases by a factor of 6x6x6=216!)

1. What is the volume of a cylinder with a 0.5m radius and a depth of 3cm? If this is a model of a bike tyre where the metal frame of the wheel has a radius of 0.45m, what is the total volume of just the tyre?

Then subtract the volume of the cylinder that is taken up by the metal frame:

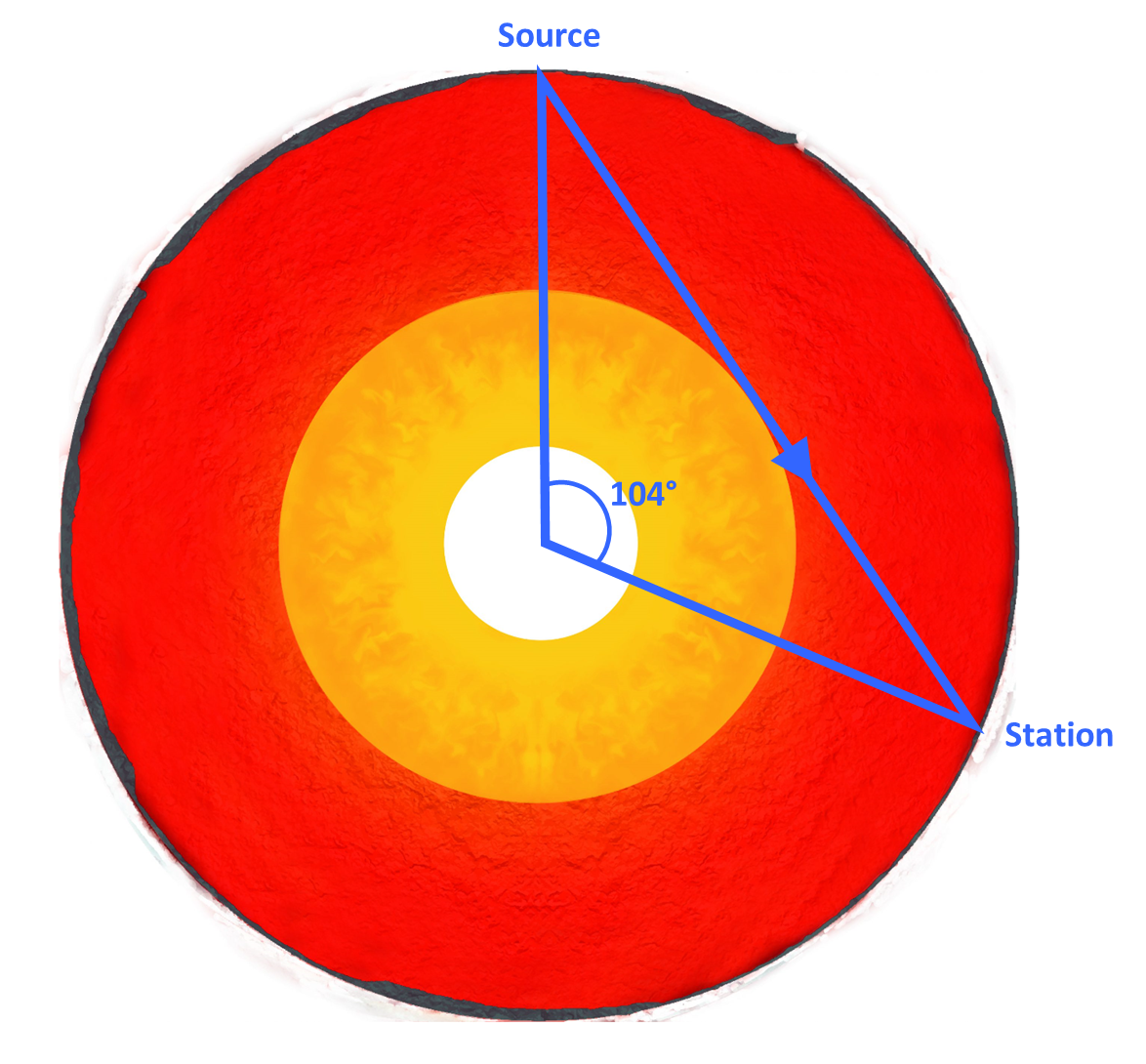
1. Earthquake energy travels through the Earth as ‘seismic waves’.

Seismic waves can be detected by seismometer stations at many different locations around the Earth due to an earthquake in a particular location.

However, none of these seismic waves travel more than 104° (measured from the centre of the Earth) round the circumference of the Earth.

This is due to the size of the core and the fact that seismic waves can’t travel through the core easily.

Use this fact to calculate the radius of the core of the Earth, given that the entire radius of the Earth is 6370km. (HARDER)

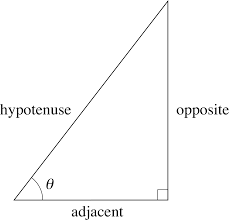


Split the blue triangle into two right angled triangles where:

The hypotenuse = 6370km, from the centre of the Earth to the Source or Station

The angle at the centre is

The shortest side is from the centre of the Earth and joins the longest blue line perpendicularly



and hypotenuse = H = 6370km and we want to find adjacent, A

Using SOHCAHTOA, we have H and A so we use cosine.

Mass, Density and Volume

Mass is the amount of ‘stuff’ you have. Volume is the amount of space that ‘stuff’ takes up. Density is the ratio of mass to volume, .

1. If a brick has the dimensions 20cm x 10cm x 10cm, calculate its volume. If this brick has a mass of 3000g, calculate its density. What are the units of density in this case?

V

1. A sphere of osmium (the highest-density metal!) has a radius of and a density of . What is the mass of this small ball of metal in kg?
2. Oil has a density of . Water has a density of . What is the overall mass of oil + water? You may have heard that oil and water don’t mix in a glass: which one (oil or water) would settle on top, and which on the bottom?

Oil sits on top since it is a lower density. Water sinks below. Even though there is a greater mass of oil in total, it is the density that matters most.

1. of sand at the surface of Earth has a mass of. of sandstone formed 500m underground has a mass of . Calculate the densities of each. Why is one denser than the other?

The pressure of 500m of sediments (sand, mud) and other rocks on top of the sandstone causes the sandstone to become more compact. This increases the density of the sandstone compared to the sand that formed it.

1. The pressure of air is related to the density of air by

At an atmospheric pressure Pa, calculate the density of the air when temperature is and when . Describe the mathematical relationship between Temperature and Density.

As temperature increases, density decreases. This is a very common occurrence! It’s why hot air rises, for instance, and why volcanoes erupt upwards, and why the Earth’s internal heat in Iceland and Yellowstone heat water deep in the Earth that travels to the surface and erupts in geysers.

Percentages and Proportions

1. There areadults in the UK who are working age. Of these, 17 million have the numeracy skills expected at KS2. What percentage is this? 42% are University graduates, 21% have A-levels and 20% have GCSEs. Calculate the number of adults in each category & estimate the ratio of graduates:A-levels:GCSEs.

(This is a true statistic!)

Ratio is 2:1:1

1. 10% of the world’s Earthquakes happen in Japan each year. If 1500 Earthquakes happen here, how many happen in the world in total? Around 90% of Earthquakes happen around the “Pacific Ring of Fire” (at the edges of the Pacific Ocean). How many Earthquakes is this per year? How many Earthquakes happen in Japan per day, and how many happen in the Pacific Ring of Fire per day? Around 25 Earthquakes are felt in the UK per year - how many days is it between each Earthquake, on average?

10% = 1500 100% = 15,000, so there are 15,000 Earthquakes globally per year.

Earthquakes in Pacific Ring of Fire

Earthquakes per day in Japan

Earthquakes in the Pacific Ring of Fire per day

between Earthquakes in the UK on average. They’re mostly very minor and we don’t feel them because of where the UK is with respect to colliding tectonic plates.

1. Due to COVID-19, joblessness claims rose by 70% to around people. What was the number of people claiming joblessness beforehand? 11% of the population are now claiming unemployment benefits in Blackpool compared to 7% before Coronavirus. There are people in Blackpool. How large is the increase in unemployed people in Blackpool due to Coronavirus?

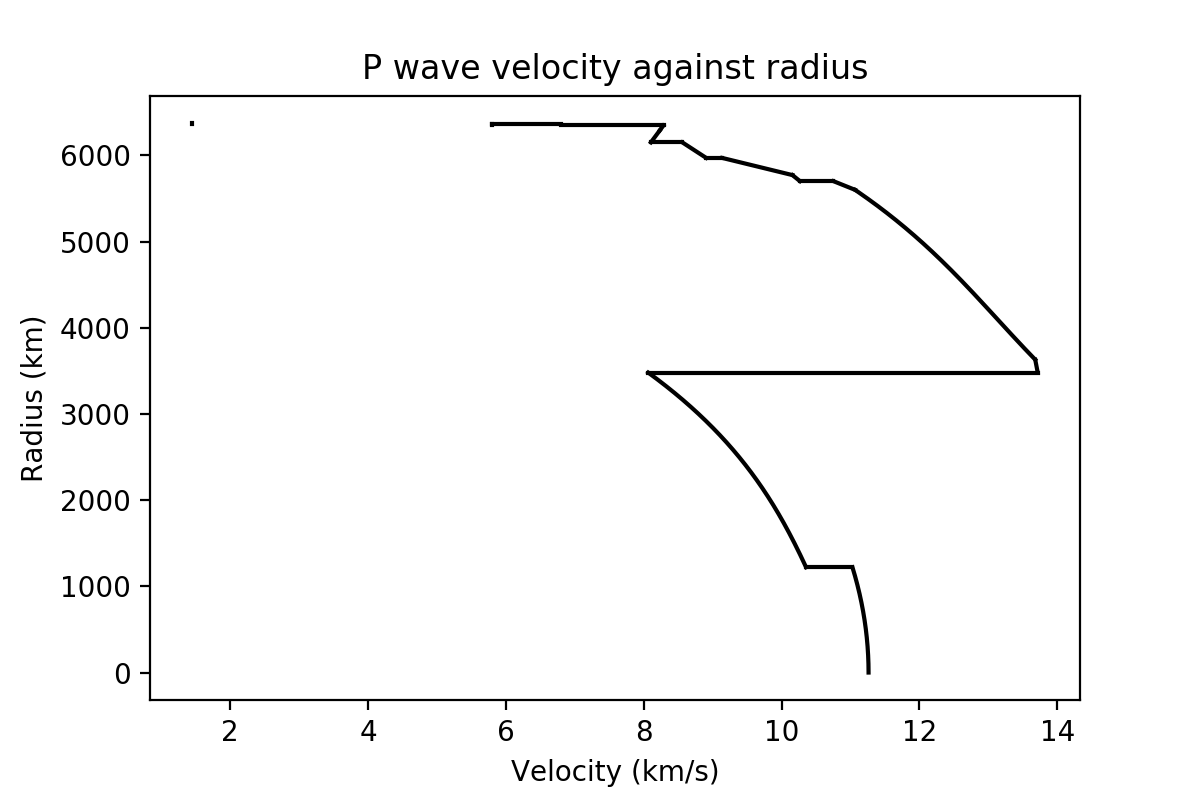
Let B = the total number of unemployed people before COVID in Blackpool

people unemployed afterwards

people unemployed before

Increase = newly unemployed people

Graph skills



1. This is a graph of some Earthquake (“P wave”) velocities through the Earth.
2. Whereabouts in the Earth do you think the “0km” and “6300km” radii are found?

0km is at the very centre of the Earth

6300km is on the surface of the Earth

1. How many distinct regions of the Earth can you make out?

3 or 4 distinct sections on the graph (0km-1100km; 1100km-3400km; 3400km-5500km; 5500km-6300km OR

0km-1100km; 1100km-3400km; 3400km-6300km)

1. What do the vertical lines mean?

Distinct regions or layers of the Earth have very different Earthquake velocities, hence the vertical ‘jumps’ at boundaries between these layers.

1. At what radius is i) the highest Earthquake speed and ii) the lowest Earthquake speed?

Highest speed just after 3400km (= 14km/s)

Lowest speed at 6300km, right at the surface (= 6km/s)

1. Describe, in a short paragraph, how you might explain the trends shown on this graph? Given that Earthquake velocities move surprisingly slowly through liquid, at what depths in the Earth is the rock likely a liquid?

The Earth is made of 3 or 4 distinct layers of material

Velocity increases generally towards the centre of the Earth, as each section curves upwards approaching 0km

Velocity is surprisingly low from 1100km-3400km, so it is probably liquid here.

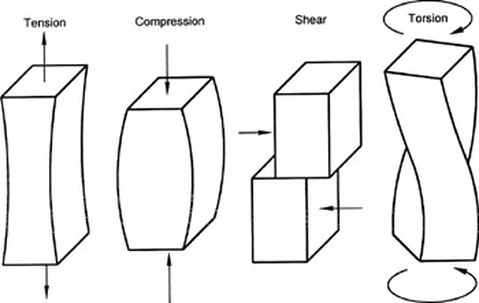
Algebra

1. Earthquake waves in the Earth come in two varieties: “P waves” and “S waves”. These have different speeds when travelling through the Earth, calculated by:

S = resistance to shearing, units: Pascals

C = resistance to compression, units: Pascals

D = density of material, units: kilograms per cubic meter ()



1. Write down expressions for and .
2. Rearrange equation (i) to find D, the density.
3. Rearrange equation (ii) to find S, the resistance to shearing
4. If ,and , calculate and . Which is faster?

P waves are faster than S waves, which makes sense from the equations (i) and (ii). This is actually always the case. The P wave velocity has an extra term added on the numerator, which makes it larger!

1. Find an expression for .
2. A liquid has a shear resistance of . (You’ll know this if you’ve ever tried to shear water - the concept doesn’t make any sense!) Write down equations for and in this case (hint: isn’t really an algebraic equation anymore, but just a number.)